Natural Class

Some guidance for the perplexed*

Charles Reiss
charles.reiss@concordia.ca

Concordia University, Montréal

Take-home messages

Rewrite rules like ‘$a \rightarrow b/c$ __ d’ are irrelevant to (intrasegmental) phonology.

- Rules can only cause addition or removal of information (valued features), so ‘→’ is misleading.
- Rules don’t refer to segments, so a, b, c are misleading—rules refer to natural classes.

Set theory refresher

a. $x \neq \{x\}$ A singleton set is different from its sole member.
b. $\{\ \} \neq$ The empty set is a set; it is not nothing.

1 What do we mean by ©Substance-Free phonology?

(1) Simple rule I: e → ẽ / __ n

(2) Simple rule II: e → i / __ n

(3) Simple rule III: e → i / __ %

(4) Simple rule IV: e → X / __ n

- There is no need to reify—or even define—notions like assimilation, dissimilation, fortition, etc.
- Environments need not provide the features to the target; the rules are computationally arbitrary in this sense.

(5) ©Substance-Freeness of Structural Changes (Dabbous et al. 2024):

The features added to segments by the application of a rule need not be found in the rule environment.

- Other work uses “substance free” with a very different meaning—the idea is that features are ad hoc language specific labels for observed patterns; features arise from patterns somehow detected in data. The “©” symbol indicates that we use the term as understood at Concordia, according to which innate features and a universal transduction system define the categories (natural classes) to which a phonology can refer; universal, innate features provide a kind of epistemic boundedness, defining the “scope and limits” of possible phonologies.
- Elsewhere, we (will) clarify that not all systematic distinctions in speech output are “phonetic”

2 Perspectives on features and substance

- Chomsky and Halle (1965, 1968): SF computation w/ universal context-sensitive transduction. No language-specific phonetics.
- Radical Substance Freeness (Fudge 1967): “The logical conclusion of this is that phonologists (above all, generative phonologists) ought to burn their phonetic boats and turn to a genuinely abstract framework. By so doing they will escape the fate of not only falling between two stools (the result of attempting to handle systematic phonemic and systematic phonetic levels in the same terms), but also ending up sitting in the very place which they have expended such strenuous and well-justified efforts to avoid”

*A nod to Maimonides (1190) and Schumacher (1977)—I have never heard of the 2001 novel.

- Poĺomská 2025:192: “The substance-free model (Chabot 2022, Iosad 2017, **Reiss 2017** [ixnay–cr], Scheer 2014, Cyran 2014) treats phonological features as abstract entities whose identity is defined by their phonological behaviour. These features are independent of phonetic content within the phonological module. The pairing of phonological features with phonetic substance takes place in the lexicon, where each language defines its own mappings.”
- Harris (1994:91): “Of course, it is always possible to profess a lack of interest in the phonetic and psychological dimensions of melodic content, in which case primes are treated as purely abstract mathematical constructs whose only function is to classify phonological contrasts in particular systems.[fn. w/ refs deleted–cr] However, the impact that Platonist views such as this have had on the recent development of phonological theory (and linguistic theory in general) has been minimal.”
- Emergent Feature Theory (Mielke 2008:9): “In emergent feature theory, features are abstract categories based on generalizations that emerge from **phonological patterns**[my emphasis-cr].”
- Cognitive Phonetics (Volenec and Reiss 2020, 2025, Reiss and Volenec 2022): SF computation w/ universal context-sensitive transduction. No language-specific phonetics.

(6) Phonetic corrlates involve disjunctions—the lack of invariance is a fact, not a problem (Appelbaum 1996):

- Phonological length: duration (kinda)—and depending on feature; burst intensity; positional variation

(7) Features and arm-raising!:

“Velar pinch” in [ak] and [ka] (Stevens 2000:p. 373). Formants pinch together, unpinch apart. Both map to +Velar.

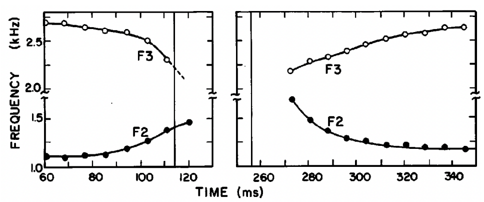


Figure 7.36 Amplitudes and frequencies of spectral peaks corresponding to some of the formants (as indicated) for the utterance /ɑkɑ/ in figure 7.35. See legend for figure 7.21. During the burst the *F2* peak is evident in the spectrum, and the amplitude of this peak is plotted as *A2*. (See, e.g., the spectrum of the burst in figure 7.35.)

3 Binarity / dichotomosity

(8) Chomsky 1957:241:

> The question of the significance of the dichotomous scale has been obscured by a certain inexplicitness in characterizing the distinctive features. A feature can be defined as a property or as a scale. In the first case (e.g., voicing, nasality), a given phone either has the property or fails to have it, just as every object is either red or nonred. Such distinctive features are thus binary by definition, and we learn nothing by being told that these features demonstrate a principle of polarity. If a feature is defined as a scale (e.g., compactness [refers to distribution of spectral energy–cr]), then we can measure each phone [...] against this scale, and ask whether there are two, three or more clusters of points along this scale. If there are two, we can say (this time significantly) that the scale is dichotomous, as a matter of fact, not definition. I have not been able to determine where exactly the authors consider the polarity to be significant (i.e., an empirical fact about clustering of points along a physically defined scale) and where it is simply a tautology. Until this is clarified, the problem of ‘binarity’ will remain confused.

(9) Chomsky and Halle 1968:384:

> If derivations such as (143) and (146) are allowed, the classificatory features are no longer binary, but rather ternary, for an unspecified feature can be distinguished from both a positively and a negatively specified feature. Hence, in a system such as this, the fact that no cost attaches to unspecified features represents a specious simplification, since lack of specification is treated as a value distinct from-and thus on a par with-the values plus and minus.

(10) For some discussion of such matters w.r.t. constructivist logic and computer science see blog entry ‘How many is two?’ at `https://math.andrej.com/2005/05/16/how-many-is-two/`:

(11) Inkelas (1995) arguing for underspecification:

“[V]irtually all objections to underspecification have actually been objections to various principles designed to regulate the distribution of this alluring device.”

“Unfortunately, however, such criticisms sometimes fail to clearly decouple the situation of underspecification from the tainted principles intended to govern it, leading to the implicit, sometimes explicit, conclusion that underspecification is equally untenable.”

- **N.B.** The existence of **bad** arguments **for** underspecification is **not** an argument **against** underspecification.

(12) Inkelas’ 1995 PROTOCOL of underspecification and Lexicon Optimization:

“In summary, Lexicon Optimization causes the underspecification of structure that is both **predictable** and **alternating**. When either condition is not met, however, Lexicon Optimization leads to the opposite scenario: full underlying specification.”

(13) Logical Phonology treats all features as scales, not properties, in Chomsky’s sense:

- **properties** are present or absent (± = present/absent)
- **scales** have values (two by hypothesis): “tall” vs. “short”. If scale is absent, there is no value: +/−/absent.
- “absent” cannot be referred to
  - not like the BLANK symbol in Turing machines
  - but like the empty string ϵ in formal language theory
- Chomsky adds to the confusion: “To clarify terminology [sic!−cr], we say that a segment has the value 0 with respect to a given feature if that feature is not assignable to it, and that otherwise it has the value + (if it has the feature) or − (if it **lacks** [my emphasis−cr] the feature).” [Chomsky 1957:236]

4 Symbol taxonomy

(14) Representation system (lexicon) (Matamoros and Reiss 2016, inspired by Gallistel and King 2009, SPE:390) needs values, features, segments, strings, etc.

(15) Computational system (rules) needs partial descriptions like natural classes, string classes, etc. These makes use of *subsumption* relation.

- UG provides $\mathscr{F} = \{F_1, F_2, \ldots F_n\}$ and $\mathbf{W} = \{+, -\}$
- $\mathbf{W} \times \mathscr{F}$ yields the set of (valued) features (e.g., +ROUND)
- Segments are *sets* of (valued) features (at least: X-slots, etc.)
- Segments are *consistent*—can’t have +F and −F (for now)
- No stipulation of *completeness*—underspecification for free:[1]
  - $\varsigma_1 = \{+F_1, -F_2, +F_3\}$
  - $\varsigma_2 = \{-F_2, -F_3\}$
- Natural classes of segments—see below

[1]OT-istas (aside from S. Inkelas) like to say “Maybe we don’t need underspecification,” based on a suggestion in a 1993 handout by Smolensky, but they use it all the time (e.g., McCarthy 2008 for debuccalization or any case of *derived surface underspecification*, as pointed out by Benz and Volenec 2023). Inkelas, working in OT, has made a strong case for the necessity of underspecification, and this work is highly influenced by hers.

5 Views about natural classes in rules

(16) Interpretive Procedure from *SPE* (Chapter 8, 337):

A rule of the form $A \rightarrow B/X\underline{\quad\quad}Y$ applies to any string $Z = \ldots X'A'Y' \ldots$, where X', A', Y' are not distinct from X, A, Y, respectively; and it converts Z to $Z' = \ldots X'B'Y' \ldots$, where B' contains all specified features of B in addition to all features of A' not specified in B.

(17) Distinctness in *SPE* (336):

'Two units U1 and U2 are distinct if and only if there is at least one feature F such that U1 is specified [αF] and U2 is specified [βF] where α is plus and β is minus...'

- The (typically implicit) appeal to subsumption in general phonological practice derives from the assumption of a logical equivalence between subsumption and non-distinctness, the idea that if x is non-distinct from y, then either x subsumes y or y subsumes x. This equivalence does not hold, however, except under the working assumption of the *SPE* era that representations are fully specified for all features, *modulo* redundancy.

(18) What happens here? [-nasal] → [-voiced]

Non-distinct representations are, in the general case, what is called (mutually) 'consistent' in unification-based frameworks—that is, they have no incompatible feature values. But non-distinctness, or consistency, does not reduce to subsumption. (See Reiss (2003), as well as Bayer and Johnson (1995) in a discussion of Lambek Categorial Grammar: "Interestingly, in cases where features are fully specified, these subsumption and consistency requirements are equivalent".)

(19) Problematic views:

a. Davenport and Hannahs (2010:91-92) conflate several issues:

- "The two sounds [[t] and [d]] can be said to constitute a natural class (of alveolar stops), in that no other sounds share this particular set of co-occurring features."
- "Natural classes may consist of any number of sounds, from two to many..." [What's the deal? See below.]
- "phonological processes typically refer to recurring groupings"
- "Non-recurring groups of sounds typically do not constitute a class"
- "We can now define the difference between 'natural' and "unnatural" classes of sounds in the following way: 'natural' classes can be specified by a single conjunction of features as in (8); 'unnatural' classes require a disjunction for their specification as in (9). We have already noted that the languages of the world appear overwhelmingly to favor natural sets of sounds in their rules. Translated in terms of feature specifications of the sort just illustrated, this means that the languages of the world prefer to deal with sets of sounds that require few specified features for their identification rather than sets that require many. If we now postulate that the rules and regularities that represent a speaker's knowledge of the phonology of his language are represented in the speaker's memory in terms of distinctive feature specifications, then this observed preference on the part of the languages of the world becomes readily comprehensible: it is but another facet of the need to conserve space in the speaker's memory, which we have already had occasion to invoke ...." [What's the big mistake?]

b. Wheeler (1972): "One function of distinctive features is to provide a formal means of expressing the notion of a NATURAL CLASS (cf. Harms, 1968: 26), such that a phonological rule which applies to a natural class of segments may be expressed in a simpler way than a rule applying to some other class of segments. Distinctive features, and notational devices and conventions, are intended to capture this notion of simplicity so that by examining a formalized rule one may discover, counting the symbols in it according to a set of values provided, at least whether the rule is simpler than a comparable rule, if not its simplicity in any absolute sense."

c. (Wheeler 1972:88): "Now the number of possible unordered sets (including sets with one member only) from a set of [Catalan's] 29 different symbols is $2^{29} - 1$, [fn. thanking his sister, Ruth appears here–cr] that is, 536,870,911. Clearly, then, any theory which can discount the vast majority of these classes as 'un- natural' has considerable power. So we want not merely a powerful theory but a correct theory of exactly the right power."

d. Hayes (2009): in rules natural classes occur "in most instances" (p. 43) or as a "pervasive tendency" (p. 71) [Confusion of intension and extension, or something more serious?]

e. Kornai (2008) assuming a language with a segment inventory that includes at least [p,t.b,d], says that "Phonologists would be truly astonished to find a language where some rule or regularity affects *p*, *t*, and *d* but no other segment"—presumably he has *b* in mind. [Hard to be astonished without a definition of rule.]

f. Lightner (1963) shows that two interpretations of rules both lead to problems on their own

g. Harms (1968:44-45) presents the *SPE* position: "The distinctiveness criterion is a condition imposed upon rules to guarantee a rigorously binary system. According to this condition no rule is well-formed (acceptable) that exploits a contrast between specified ('+' or '-'), and unspecified ('0'), features. In effect, for all segments possibly relevant to the structural description of a given rule, there must be a way of determining solely on the basis of 'plus' and 'minus' feature values whether or not the rule is to apply. ...The distinctiveness criterion thus requires that each phoneme contrast with every other phoneme in at least one specified feature. If this were not the case, no rules could be given for filling redundant feature values."

h. Harms (1968:26) "It is a class of segments that can be specified with fewer features than any individual member of the class." [He is forgetting about singleton sets, e.g. {t} vs. {t, d}.]

i. Halle and Clements (1983:9) go for the subsumption idea: "We can now define the difference between "natural" and "unnatural" classes of sounds in the following way: "natural" classes can be specified by a single conjunction of features [...]; "unnatural" classes require a disjunction for their specification [...].

"We have already noted that the **languages of the world appear overwhelmingly to favor natural sets of sounds in their rules**. Translated in terms of feature specifications of the sort just illustrated, this means that the languages of the world prefer to deal with sets of sounds that require few specified features for their identification rather than sets that require many [Why is this patently false?–cr]. If we now postulate that the rules and regularities that represent a speaker's knowledge of the phonology of his language are represented in the speaker's memory in terms of distinctive feature specifications, then this observed preference on the part of the languages of the world becomes readily comprehensible: it is but another facet of the need to conserve space in the speaker's memory, which we have already had occasion to invoke..."

(20) Unobjectionable views that say just enough:

a. "central claim" that targets, environments *are* natural classes defined intensionally via "conjunctions of properties" (e.g., Odden 2013:159)

b. "concept of a natural class allows us to sharpen up what we mean by a phonological rule" (Spencer 1996:135)

6 Natural classes in Logical Phonology

- No concern with minimizing redundancy
- No concern with transderivational relations among segments (as a concern of the grammar or the learner)
- No concern with phonemicity as discussed by Harms (1968), Lightner (1963)

(21) Two feature system—let's ignore the complexities of "real" languages:

- $\mathscr{F} = \{ \text{F, G} \}$
- $\mathbf{F} = \mathscr{F} \times V = \{ -\text{F}, -\text{G}, +\text{F}, +\text{G} \}$
- $\mathscr{I}$ = consistent subsets of **F**
- Three-way choices for F and G: +, –, 'absent' $\rightsquigarrow 3^2 = 9$ segments

{	{ }	{−G}	{+G}	{−F}	{−F, −G}	{−F, +G}	{+F}	{+F, −G}	{+F, +G}	}
	0	1	2	3	4	5	6	7	8	
	0+0	0+1	0+2	3+0	3+1	3+2	6+0	6+1	6+2	
	FG	FG	FG	FG	FG	FG	FG	FG	FG	
	$3^1 3^0$	$3^1 3^0$	$3^1 3^0$	$3^1 3^0$	$3^1 3^0$	$3^1 3^0$	$3^1 3^0$	$3^1 3^0$	$3^1 3^0$	
	00	01	02	10	11	12	20	21	22	

(22) What are the natural classes? Classes defined by (consistent) conjunctions of valued features.:

	Natural class	Extension
0	[ ]	{ς0, ..., ς8}
1	[−G]	{ς1, ς4, ς7}
2	[+G]	{ς2, ς5, ς8}
3	[−F]	{ς3, ς4, ς5}
4	[−F, −G]	{ς4}
5	[−F, +G]	{ς5}
6	[+F]	{ς6, ς7, ς8}
7	[+F, −G]	{ς7}
8	[+F, +G]	{ς8}

(23) Nothing else is a natural class:

- Only a fully specified segment corresponds to a singleton natural class: {t} but not {D}
  - The class [+F, +G] = $[\textcircled{\varsigma 8}]$ = $\{\varsigma 8\}$ contains only the segment {+F, +G} = $\varsigma 8$. So, Harms was wrong in (19i).
  - The class [+F] = $[\textcircled{\varsigma 6}]$ = $\{\varsigma 6, \varsigma 7, \varsigma 8\}$ contains the segments {+F}, {+F, −G}, {+F, +G} = $\varsigma 6, \varsigma 7, \varsigma 8$
  - $\{\varsigma 6\}$ is not a natural class
- $\{\varsigma 3, \varsigma 4, \varsigma 5\}$ is a natural class: {t, D, d}
- $\{\varsigma 4, \varsigma 5\}$ is *not* a natural class: {t, d}
- $\{\varsigma 3, \varsigma 4\}$ is *not* a natural class: {t, D}
- The empty set of segments is *not* a natural class—so, we can't use natural class notation to replace boundary symbols. We can't write, say, "before no segment" without a boundary symbol.

(24) Formalization of pretty traditional notion of natural class applied to universal feature set:

A subset P of the universal segment inventory $\mathscr{I}$ is a natural class iff there exists a consistent feature set $\mathscr{C}$ such that $P = \{p : p \supseteq \mathscr{C}\}$

(25) Partial description for "high front vowels":

- Set builder notation: $\left\{x : x \supseteq \left\{\begin{matrix} +\textsc{Syllabic} \\ +\textsc{High} \\ -\textsc{Back} \end{matrix}\right\}\right\}$
- Square brackets: $\begin{bmatrix} +\textsc{Syllabic} \\ +\textsc{High} \\ -\textsc{Back} \end{bmatrix} \neq \left\{\begin{matrix} +\textsc{Syllabic} \\ +\textsc{High} \\ -\textsc{Back} \end{matrix}\right\}$
- Circle notation removes a set of curly brackets: $[\textcircled{\mathscr{U}}]$ (regardless of whether $/\mathscr{U}/$ is 'in the language').
- For a fully specified segment $/\phi/$, $[\textcircled{\phi}] = \{x : x \supseteq \{\textcircled{\phi}\}\} = \{\phi\}$

(26) A natural class vs. the manifestation/projection/instantiation of a natural class in L_i:

- "In English {p, b, m} is a natural class but {p, m} is not, however in Cree {p, m} is a natural class."
- I suggest that only the universal intensional notion of natural class is relevant—that is why generalizations exists. If the inventory mattered, there would be no generalization.
- Inventory is defined on the surface, but rules don't apply on the surface, so the surface inventory can't be relevant. /d/ -> D -> t is typical. If we are correct about feature changing then inventories become even less relevant.

(27) Unnatural classes:

There are $2^9 = 512$ sets of (consistent) segments (the members of the Powerset of the segment set) but only the 9 shown above are natural classes. So there are 503 unnatural classes. These cannot appear in rules, which are intensionally defined. We have a theory and it is precise.

(28) Relativized to a level of representation:

- Lexical segments? Surface segments?
- Given, say, /d/ → D → [t] what is the natural class of coronal stops in this language?
- Just do everything intensionally.

- $x \neq \{x\}$: A singleton natural class is not a segment, it is a set. Watch N.C. saying this at https://youtu.be/GPHew_smDjY?t=324.

(29) **Specificity: The Phonological Sawzall** (after Lees 1961):

Let f, g be segments (feature specifications) such that $f \subset g$. Then there is no natural class containing f but excluding g.

(30) We now have a partial theory of what can and cannot be the targets and triggers of rules, of the "scope and limits" of phonology. We will see that this formalization of natural classes allows our austere model to expand the scope of phonology beyond what is traditionally believed to be the case.

7 Parts of rules

(31) Notational / terminological note—we take brackets seriously:

yes: The vowel /i/ is (specified) +High   **no:** The vowel /i/ is (specified) [+High].

Note then that traditional rule notation is not type-consistent:

- a rule's target and environment are natural classes,
- but the change is a feature specification.

(32) Extensional formulation:

o, e	→	u, i	/ __	m, n, ŋ
(Target)		(Change)		(Environment)

(33) Traditional formulation:

[−Low]	→	[+High]	/ __	[+Nasal]
(Target)		(Change)		(Environment)

(34) Revised LP formulation:

[−Low]	→	{+High}	/ __	[+Nasal]
(Target)		(Change)		(Environment)

(35) A theory of natural classes is part of a theory of rules:

- If /ɪ,e/ do X, then we predict that /i/ will also do X, even in the absence of evidence to the learner.
  - i is subsumed by ɪ∩e
  - $/\text{i}/ \supseteq \left\{ \begin{array}{l} -\textsc{Back} \\ -\textsc{Low} \\ -\textsc{Round} \end{array} \right\}$ or equivalently $/\text{i}/ \in \left[\begin{array}{l} -\textsc{Back} \\ -\textsc{Low} \\ -\textsc{Round} \end{array} \right]$
  - and ɛ? and æ?
  - Other, (non-©, so-called "Radical") Substance Free Phonology models seems to predict that anything goes. Theories that make no predictions have limited utility.

(36) **Quiz**: Which of the following are ways to begin a rule? i. d →   ii. [d] →   iii. [ⓓ] →   iv. [Ⓓ] →

8 Operator 1: Set subtraction removes features

- The → symbol has several different uses including changing, inserting, deleting features and segments
- LP deconstructs → into a system of three operators.
- Deconstruction precludes the need for a distinction between feature-filling and feature-changing rules.

(37) Subtraction: $A \setminus B = \{c\text{F} \mid c\text{F} \in A \wedge c\text{F} \notin B\}$

- To formulate subtraction rules, we assume:

    - the **target is a natural class**, appearing on the left-hand side of the subtraction (so, not, say, /e/)
    - the change is a feature specification, appearing on the right-hand side,
    - an optional environment is specified using natural classes.

(38) [−Back] ∖ {−High} / ___ [+Nasal]

(39) Anathema to Harms (1968:44-5) on account of distinctness and removal of features:
Features cannot be deleted from within a segment: nor could this ever be desirable, for reasons which should be obvious in light of the distinctiveness criterion discussed below [i.e. above in (19f)–cr].

You can have missing features if they are redundant, but you can't delete!

- $X \setminus Y$ is set Z containing all elements of X not in Y
    - {a, b, c} ∖ {a, b, d} = {c}
    - {+Voiced, +Coronal, −Son, −Continuant } ∖ {+Voiced}

      = {+Coronal, −Son, −Continuant }
    - Subtraction implements feature *removal*, like "delinking" in autosegmental models

(40) $\begin{Bmatrix} +\text{High} \\ -\text{Round} \\ -\text{Back} \end{Bmatrix} \setminus \begin{Bmatrix} +\text{High} \\ +\text{Round} \end{Bmatrix} = \begin{Bmatrix} -\text{Round} \\ -\text{Back} \end{Bmatrix}$ (This is *not* a rule—why not?)

(41) Arbore debuccalization (Benz and Volenec 2023, McCarthy 2008, Hayward 1984):

Glottalized obstruents ⇝ ʔ in Coda

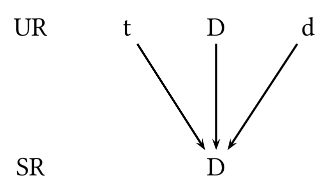
/beːkˀ.taw/ ⇝ [beːʔ.táw]

- subtract all the Place features—*derived surface underspecification*
- Rule: [+Glottal] ∖ {+Coronal, −Coronal, +Labial, ...} in Coda
- Interpretation:[2] subtract that set of features from any member of the set of consonants that have +Glottal as a member.

(42) Subtraction in action:
- /d/ ∖ {+Voice} = /D/
- Vacuous subtraction: /d/ ∖ {−Voice} = /d/
- /d/ ∖ {−Voice, +Voice} = /D/

Neutralization of /t, D, d/ in some context.

- [(D)] ∖ {−Voice, +Voice} ⇝ [D], [D], [D]



/D/ is unaffected because subtraction is vacuous. We are not targeting " αVoiced" but rather the members of the extension of a partial description that does not mention Voiced

[2]See discussion of rule interpretation in Bale and Reiss (2018)—rules actually maps strings to strings (or the equivalent for other structures), but for our purposes here it is useful to treat rules as mapping segments to segments.

QUIZ—Specificity Review: What is the rule target and subtrahend set in each case of a rule removing VOICED specifications? Use the circle notation. There may be more than one possible answer—give them all.

a. L_1 has only underlying {t, D} and only /t/ is affected—/t/ ⇝ [D].

b. L_2 has only underlying {d, D} and only /d/ is affected—/d/ ⇝ [D].

c. L_3 has only underlying {d} and /d/ is affected—/d/ ⇝ [D].

d. L_4 has only underlying {t, d}, and only /d/ is affected—/d/ ⇝ [D].

9 Operator 2: Unification for feature-filling rules

The *unification* operator ⊔, adapted from syntactic theory (e.g., Shieber 1986), adds a valued feature to a segment, but has no effect if the target is already specified for that feature.

(43) Unification: $A \sqcup B = A \cup \{c\mathrm{F} \mid c\mathrm{F} \in B \wedge -c\mathrm{F} \notin A\}$

In other words, $A \sqcup B$ contains all features of A as well as any features of B which do not conflict with those in A. Thus unification only adds valued features when the target (the first argument) is underspecified with respect to the features in the change, as illustrated below. Unification is thus a solution to SPECIFICITY, i.e., the problem of targeting underspecified segments, and corresponds to so-called *feature-filling* (or *structure-building*) processes.

9.1 Turkish three-way voicing contrast

- There are three kinds of plosive-final roots in Turkish.

(44) Ternary voicing (Inkelas 1995):

		nom.sg.	acc.sg.	nom.pl.	1sg.poss.	
a.	voiceless:	sana**t**	sana**t**-ɨ	sana**t**-lar	sana**t**-ɨm	‘art’
b.	voiced:	ety**d**	ety**d**-y	ety**d**-ler	ety**d**-ym	‘etude’
c.	alternating:	kana**t**	kana**d**-ɨ	kana**t**-lar	kana**d**-ɨm	‘wing’

Inkelas proposes final plosives in roots like (44), are underspecified for VOICE.

Forms like [edʒdat- / edʒdad-] ‘ancestry’ from /edʒdaD-/, with a non-alternating voiced /dʒ/ and alternating /D/ show that the pattern cannot be attributed to a lexical exception feature.

(45)

	/t/	/d/	/D/
CONTINUANT	−	−	−
VOICE	−	+	

She proposes that the processes which fill in voice specifications are strictly feature-filling

LP does not make this distinction, and so it affects /D/ without affecting /t, d/ via vacuity.

(46) Onset voicing: $[-\textsc{Continuant}] \sqcup \{+\textsc{Voice}\} \ / \ [_\sigma \_\_$

(47) Yield of (46):

a.	/t/ ⊔ {+VOICE} ⇝ /t/	(unification failure)
b.	/d/ ⊔ {+VOICE} ⇝ /d/	(vacuous unification)
c.	/D/ ⊔ {+VOICE} ⇝ /d/	(feature filling)

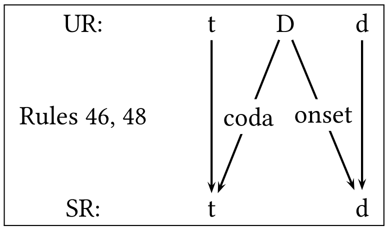
- Parallel reasoning applies to Coda devoicing

(48) Coda devoicing: $[-\textsc{Continuant}] \sqcup \{-\textsc{Voice}\} \ / \ \_\_ \,]_\sigma$

- Again, only application to /D/ is non-vacuous. Onset /D/ ⇝ [d], but /t/ is inalterable.

- Turkish appears complex, requiring "exceptionality" but the LP analysis is simple—involving just unification and underspecification (which comes for free).

(49) Turkish segment mapping diagram (crucial rule ordering ignored here):



9.2 Inkelas's use of underspecfication

- The behavior of /D/ fulfills both conditions (**predictable** and **alternating**), and the three-way consonant split forces the analysis (barring diacritic/diacritic solutions).
- As we proceed, we will see that Inkelas' protocol must be rejected.

9.3 Baztan Basque—reduction to pure phonology, again

- Hualde (1991) discusses Low Vowel Assimilation (henceforth, LVA) in several dialects of Basque. LVA is attributed to "a rule that raises a low vowel to /e/ after a high vowel, with or without any intervening consonants" (op. cit.:23).

(50) Low Vowel Assimilation (loc. cit.):

/gogo-a/	[gogoa]	'the spirit, will'
/buru-a/	[burue]	'the head'
/mutil-a/	[multil**e**]	'the boy'
/mendi-a/	[mendi**e**]	'the mountain'
/egun-a/	[eɣun**e**]	'the day'

- We focus on Hualde's account of the Baztan dialect (§2.2), in which LVA affects vowels within words and across certain word boundaries.

(51) Auxiliary verbs with raised vowel after high vowel (op. cit.:29–30.):

a.	torri d**e**	'he has come'	cf. gan d**a**	'he has gone'
b.	gain d**e**	'he will go'	cf. lorriko d**a**	'he will come'
c.	torri g**e**ra	'we have come'	cf. gan g**a**ra	'we have gone'
d.	torri z**e**ra	'you have come'	cf. gan z**a**ra	'you have gone'
e.	in z**e**zu	'do it'	cf. jan z**a**zu	'eat it'

- Hualde claims morphemes in which /a/ are unaffected by LVA belong to different morphological strata from those where it does, but it seems difficult to maintain this position.

(52) Exceptions to LVA (op. cit.:26–30):

- "Not all derivational suffixes present the same behaviour with respect to Low Vowel Assimilation."
- "Verbal suffixes generally undergo assimilation."
- "We need to examine now the application of the rule in conjugated verbal forms. Here, the situation is not uniform… [I]n quite a few conjugated forms the rule fails to apply."
- "The forms that present the context for Low Vowel Assimilation, but, nevertheless, do not undergo the rule, have just one more irregularity that must be lexically marked."
- "Certain auxiliary verbs also undergo assimilation… Other forms of the auxiliary, on the other hand, never undergo assimilation, including the other persons of the intransitive present indicative not mentioned above…"

  – “We must conclude that only a few auxiliary forms can behave like clitics and thus undergo Low Vowel Assimilation.”

- It is hard to imagine any account which would unify the morphosyntactic “exceptions”
- We instead propose that some morphemes contain underspecified /A/ which is *mutable* via a unification rule (e.g., the singular definite /-A/, the auxiliary /dA/), whereas others (e.g., the auxiliary /naiš/) contain a prespecified /a/ which is *inalterable* with respect to this rule.

(53) Baztan vowel specification (partial):

	/a/	/e/	/A/	/o/
High	–	–	–	–
Low	+	–		–
Back	+	–		+
Round	–	–	–	+

(54) Low Vowel Assimilation: $\begin{bmatrix} -\textsc{High} \\ -\textsc{Round} \end{bmatrix} \sqcup \{-\textsc{Low}\} \;/\; [+\textsc{High}]\ C_0\ \_\_$

(55) Yield of (54):

a.	/a/ $\sqcup \{-\textsc{Low}\} \rightsquigarrow$ /a/	(unification failure)
b.	/e/ $\sqcup \{-\textsc{Low}\} \rightsquigarrow$ /e/	(vacuous unification)
c.	/A/ $\sqcup \{-\textsc{Low}\} \rightsquigarrow$ /e/	(feature filling)
d.	/o/ $\sqcup \{-\textsc{Low}\} \rightsquigarrow$ /o/	(vacuous unification)

- It only *seems* like (54) targets the underspecified segment to the exclusion of others
- Turkish and Basque show the contrast between *inalterable* (prespecified) segments and *mutable* (underspecified) segments w.r.t. unification rules.
- Inkelas’ Protocol still looks tenable: /A/ alternates and its surface forms are predictable

10 Combining subtraction and unification

Unification makes feature-filling easy—solving a longstanding problem. Since we also have subtraction, who needs feature-changing rules?

10.1 Russian

Following the suggestions of Poser (1982), Inkelas and Cho (1993), and Siptár and Törkenczy (2000) we model feature-changing processes as subtraction followed by unification: d $\rightsquigarrow$ D $\rightsquigarrow$ t.

(56) Russian final devoicing:

	nom.sg.	gen.sg.	
a.	t͡svʲet	t͡svʲeta	‘color’
b.	prut	pruda	‘pond’

(57) Consonant features:

	/t/	/d/	/D/
Voice	–	+	
Sonorant	–	–	–

(58) Part 1 (deletion): $[-\textsc{Sonorant}] \setminus \{+\textsc{Voice}\} \;/\; \_\_\ \%$

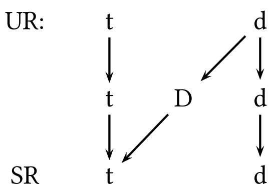
- Rule (58) maps any word-final segment-set that is a superset of {–Sonorant} to that same segment-set minus {+Voice}.

(59) Part 2 (insertion): $[-\textsc{Sonorant}] \sqcup \{-\textsc{Voice}\}$

(60) Yield of (59):

a.	/t/ $\sqcup \{-\textsc{Voice}\} \rightsquigarrow$ /t/	(vacuous unification)
b.	/d/ $\sqcup \{-\textsc{Voice}\} \rightsquigarrow$ /d/	(unification failure)
c.	/D/ $\sqcup \{-\textsc{Voice}\} \rightsquigarrow$ /t/	(feature filling)

(61) Russian segment mapping diagram—targeting of /D/ is illusory:



10.2 Hungarian reciprocal neutralization

- Hungarian exhibits a more complex process of reciprocal voice neutralization.

(62) Hungarian reciprocal voicing (Siptár and Törkenczy 2000:§4.1.1):

	nom.sg.	iness.sg	dat.sg.	abl.sg.	
a.	kalap	kala[b]-ban	kalap-nak	kalap-tól	‘hat’
	rés	ré[z]-ben	rés-nek	rés-től	‘slit’
	zsák	zsá[g]-ban	zsák-nak	zsak-tól	‘bag’
	kút	kú[d]-ban	kút-nak	kút-tól	‘well’
b.	rab	rab-ban	rab-nak	ra[p]-tól	‘captive’
	víz	víz-ben	víz-nek	ví[s]-től	‘water’
	meleg	meleg-ben	meleg-nek	mele[k]-től	‘warmth’
	kád	kád-ban	kád-nak	ká[t]-tól	‘tub’
c.	szem	szem-ben	szem-nek	szem-től	‘eye’
	őr	őr-ben	őr-nek	őr-től	‘guard’

- In this language obstruents take on the voicing of the following obstruent, which might be written as follows in SPE notation (with our brackets). Why can't LP do this?

(63) Reciprocal voice neutralization (traditional notation, to be revised):

$$\left[-\textsc{Sonorant}\right] \rightarrow \{\alpha\textsc{Voice}\} \;/\; \_\_ \begin{bmatrix}-\textsc{Sonorant}\\ \alpha\textsc{Voice}\end{bmatrix}$$

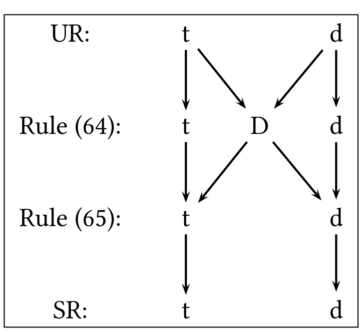
- We now proceed to decompose this into a two-step process.

(64) Part 1 (deletion): $\left[-\textsc{Sonorant}\right] \setminus \{\alpha\textsc{Voice}\} \;/\; \_\_ \begin{bmatrix}-\textsc{Sonorant}\\ -\alpha\textsc{Voice}\end{bmatrix}$

(65) Part 2 (insertion): $\left[-\textsc{Sonorant}\right] \sqcup \{\alpha\textsc{Voice}\} \;/\; \_\_ \begin{bmatrix}-\textsc{Sonorant}\\ \alpha\textsc{Voice}\end{bmatrix}$

(66) Notes: Critical ordering: (64) ≪ (65); α's in the rules are independent—we could have used α_1 and α_2

(67) Hungarian segment mapping diagram:



- Note that we were able to use the same two operations (abstractly, $\phi \rightarrow \Delta$ and $\Delta \rightarrow \phi$) for both Russian and Hungarian even though reciprocal voicing "looks" more complex than final devoicing.

(68) Inkelas' protocol needs elaboration:

Russian and Hungarian show how **subtraction can make inalterable segments become mutable**. Underspecification can underlying as in Turkish, or be derived, as in Russian and Hungarian. Despite the underlying segments like /t/ and /d/ alternating and surfacing in predictable ways, there is **no underspecification in the lexicon. Lexicon Optimization in Inkelas' sense is irrelevant.**

10.3 Greek fortition

			IPFV.1SG /-o/	PFV-1SG /-s-o/	
(69)	a.	γρά**φ**-	[ˈɣra**f**o]	[ˈɣra**p**so]	'write'
	b.	ρά**β**-	[ˈɾa**v**o]	[ˈɾa**p**so]	'sew'
	c.	βραβε**ύ**-	[vraˈve**v**o]	[vraˈve**f**so]	'award'

Gorman (to appear): *inalterable* υ is underlyingly +CONTINUANT, whereas mutable φ, β are underspecified for this feature.

(70) Feature specification (partial):

		CONTINUANT	VOICE
/F/	⟨φ⟩		−
/V/	⟨β⟩		+
/v/	⟨υ⟩	+	+

(71), a unification rule strengthens /F, V/ when followed by a sibilant, but applies vacuously due to unification failure to inalterable /v/. A later context-free redundancy rule specifies /F, V/ not targeted by (71) as fricatives. For completeness, Gorman also provides rules for the feature-changing voice assimilation process and sample derivations.

(71) Fortition: [+LABIAL] ⊔ {−CONTINUANT} / __ [+STRIDENT]

(72) Default frication: [+LABIAL] ⊔ {+CONTINUANT}

(73) Voice assimilation (part 1: deletion): [−SONORANT] \ {αVOICE} / __ [αVOICE]

(74) Voice assimilation (part 2: insertion): [−SONORANT] ⊔ {αVOICE} / __ [αVOICE]

(75) Sample derivations (stress omitted):

UR	/ɣraF-o/	/ɣraF-s-o/	/ɾaV-o/	/ɾaV-s-o/	/vravev-o/	/vravev-so/
Rule (71):		ɣra**p**so		ɾa**b**so		
Rule (72):	ɣra**f**o		ɾa**v**o			
Rules (73–74):				ɾa**p**so		vrave**f**so
SR	[ɣrafo]	[ɣrapso]	[ɾavo]	[ɾapso]	[vravevo]	[vravefso]

(76) Inkelas' protocol fails again:

Note that /v/ alternates and its surface forms are predictable. Yet we do not posit underspecification. On the other hand, /V/ alternates and its surface forms are also predictable, but here we need underspecification. Inkelas restricted her reasoning to a single feature, whereas here we have to pay attention to both CONTINUANT and VOICE. The fully specified /v/ alternates with respect to VOICE, but it is immune to the effects of rule (73) because of unification failure. Underspecification does not reflect Lexicon Optimization in Inkelas' sense, but rather the fact that "the relation between a phonemic system and the phonetic record …is remote and complex" (**?**: 38)…

11 Dealing with the denial of human nature: Get the sawzall

(77) A "phonologically active L-shaped class" in Kinyamwezi (Mielke 2008:131):

These three vowels glidify instead of assimilating to a following (non-identical) vowel.[3]

[3] Mielke's discussion is actually at odds with the data he presents in several ways, but the example is useful for illustration.

i		u
ɪ		ʊ
e		o
	a	

(78) **Quiz**: Posit some features and work on a rule to make only some vowels become glides

- Inkelas' protocol fails: /I/ does not alternate, but it must be **underspecified to avoid being a target of the subtraction rule**. The underspecification is driven by the *absence* of alternation.

12 Extending specificity to triggers

12.1 Barrow

- Barrow Inupiaq has three surface vowels: [i, a, u]. As discussed by Archangeli and Pulleyblank (1994:§2.2.2), Buckley (1994), and Dresher (2009:§7.2.1), among others:
  - "strong" *i*'s are catalytic and trigger palatalization of a following coronal,
  - but "weak" *i*'s (< Eskimo-Aleut *ə) are quiescent and do not.

(79) Palatalization (Kaplan 1981:§3.22):

a.	iki	'wound'	iki-**ʎ**u	'and a wound'	iki-**ɲ**ik	'wounds'
b.	ini	'place'	ini-**l**u	'and a place'	ini-**n**ik	'places'

- We propose that weak *i* as in *ini* is underspecified relative to strong *i* as in *ik**i***, and derive palatalization via a two-step subtraction-and-unification process.

(80)

	strong *i*	weak *i*	a	u
High	+	+	−	+
Back	−		+	+

(81)

	plain coronals	palatal coronals
Anterior	+	−
Coronal	+	+

(82) $[+\textsc{Coronal}] \setminus \{+\textsc{Anterior}\} \;/\; \begin{bmatrix}+\textsc{High}\\ -\textsc{Back}\end{bmatrix}$ — (strong *i* but not weak *i* is a trigger)

(83) $[+\textsc{Coronal}] \sqcup \{-\textsc{Anterior}\}$ (feature-filling palatalization)

(84) $[+\textsc{High}] \sqcup \{-\textsc{Back}\}$ (neutralization of strong *i* and weak *i*)

(85) Critical ordering: (82) ≪ (83–84)

(86) Insufficiency of Inkelas's Lexicon Optimization approach to underspecification:

- Strong *i* is *catalytic*—natural classes can specify it
- Weak *i* is quiescent—natural classes can exclude it by specifying what it lacks
- Weak *i* **does not alternate, but we are still licensed to posit underspecification**, apparently.

(87) Interaction matters:

The underlying identity of a segment is determined (by learner and analyst) not only by the segment's surface forms, but also by its effects (on other segments, on stress, whatever).

12.2 Salentino

(88) Francavilla–Fontana Salentino adjectives (Calabrese 1985:11):

	sg.	pl.	
fem.	ˈfredda	ˈfr**e**ddi	‘cold’
masc.	ˈfr**i**ddu	ˈfr**i**ddi	

Exactly like Barrow—just because there *appears* to be a generalization (w.r.t. gender), we don’t assume that it is grammatically relevant. Phonological accounts take priority—we assume that this is just how humans are built.

12.3 Cervara “extra-catalytic” vowels

(89) Cervara nominals (op. cit.:508–509):

a.	fr**e**lˈl**e**nka	‘vulva (vulgar)’	fr**i**lˈl**i**nku	‘penis (vulgar)’
b.	ˈbːɛlle	‘beautiful (fem. pl.)’	ˈbː**e**ʎʎi	‘beautiful (masc. pl.)’

Cervara doesn’t show any quiescent surface high vowels; instead it has “extra-catalytic” ones. The high vowel ending *i* in verbs affects preceding mid vowels, as we saw in nouns, but it raises even the lax ones to become +High:

(90) Cervara verbs (op. cit.:514):

a.	ˈv**e**do	‘I see’	ˈv**i**di	‘you see’
	ˈv**e**de	‘he sees’	ˈv**i**du	‘they see’
b.	ˈmɛto	‘I reap’	ˈm**i**ti	‘you reap’
	ˈmɛte	‘he reaps’	ˈm**i**tu	‘they reap’

12.4 Taxonomy

- The following summarizes possible interactions between under-/prespecified segments.
  - Let the phonemic inventory be /x, y, X, Y/.
  - Suppose /x/ is (pre)specified as $+x$ and the underspecified segment /X/ = /x/ $\setminus \{+x\}$
  - Similarly, suppose that /y/ is (pre)specified as $+y$ and the underspecified segment /Y/ = /y/ $\setminus \{+y\}$
  - Assume rule: [ⓧX] $\sqcup \{-x\}$ / __ [ⓨy]
  - Applying rule to segment /X/ yields /w/: /X/ $\sqcup \{-x\}$ =/w/

(91) Interaction taxonomy:

a.	x / __ Y:	inalterability	× quiescence	⇝	no effect
b.	x / __ y:	inalterability	× catalysis	⇝	no effect
c.	X / __ Y:	mutability	× quiescence	⇝	no effect
d.	X / __ y:	mutability	× catalysis	⇝	effect: X ⇝ w

- In the next section, will see a language that illustrates the full taxonomy in (91).

12.5 Blackfoot

- Frantz (2017: ch. 6) describes a breaking process in Blackfoot:

> The *s* following the future prefix in [*kit-áak-***s***-ipii* ‘you will enter’] requires some discussion. The initial vowel of stem *ipii* ‘enter,’ unlike the initial vowel of *itsiniki* ‘tell a story,’ always causes a preceding *k* to be replaced by the affricate *ks*. We will speak of this phenomenon as breaking of *k*, and of the *i* which is involved as a breaking *i*. For any morpheme which begins with *i* we need to know whether that i is a breaking *i* or not; if it is a breaking *i*, then if it immediately follows a morpheme ending in *k* we know that the *k* will be replaced by *ks*. (82–83)

- But there is one wrinkle: the second person prefix *k*- is "always impervious to breaking" (93). Breaking also seems to require a distinction between inalterable and mutable targets.

(92) Blackfoot derivations:

UR	ki	kI	Ki	KI
/K/ breaks before /i/:			$\widehat{\text{ks}}$i	
Otherwise /K/ to [k]:				kI
/I/ to [i]:		ki		ki
SR	ki	ki	$\widehat{\text{ks}}$i	ki

We can derive the Blackfoot pattern with just three unification rules.

12.6 Hungarian complications

12.6.1 Quiescent and mutable: Hungarian v_1 = /V/

12.6.2 Catalytic and inalterable: Hungarian h_1 = /H/

(93) Summary of Hungarian complications w.r.t. Inkelas' protocol:

/V/ is mutable to start but the others have to be made mutable. In the meantime, /V/ fails to be catalytic. /H/ is inalterable by voicing rule, because prespecified, but it does not become mutable (like the others do, because it lacks a different feature, +Consonantal!) This goes beyond Inkelas's model of underspecification: H does not alternate w.r.t. voicing, but it is underspecified for some other feature, with respect to which it does not alternate.

13 Operator 3: Segment insertion and deletion rules

- The remaining operator is the *segment* operator ↦, combined with a null symbol ϵ, implements insertion and deletion of full segments.[4]

(94) Insertion schema:

$$\epsilon \;\mapsto\; \overset{\text{X}}{\underset{\{\ldots\}}{|}} \;/\; \ldots$$

(95) Deletion schema:

$$\overset{\text{X}}{\underset{[\ldots]}{|}} \;\mapsto\; \epsilon \;/\; \ldots$$

- In insertion rules, the X is linked to a feature bundle and a specific segment is inserted.
- In deletion rules, however, the X is linked to a natural class (partial description) and any segment matching the natural class is deleted, and thus the targets of deletion rules must conform to Specificity.

(96) Delete The Rich (Reiss 2025): If some—but not all—X's delete in some phonological context Y __ Z, the X's that delete must be **more richly specified** than those which do not.

- With unification rules, the Specificity effect is concealed by vacuous application and unification failure, but there is no such escape hatch for deletion rules.[5]
  - A unification rule of the form [Ⓘ] ⊔ {+High} will apply non-vacuously only to /I/.
  - However, a deletion rule targeting [Ⓘ] both intensionally and extensionally targets /i, e, I/.

[4] Note that we do not model segment deletion via subtraction. Subtraction removes features, not segments, and we do not equate segmental underspecification with segmental absence. Removing all the features of a set leaves a set, the empty set; it does not leave nothing.

[5] An anonymous reviewer asks whether one could decompose segment deletion into subtraction followed by deletion of the X. However, Delete The Rich entails the second rule—deleting only those X's linked to a null feature bundle—is unstatable in LP.

- Delete The Rich follows directly from our definitions, but runs contrary to a common intuition that less richly specified segments are "weaker" and thus are preferential targets for deletion (e.g., van Oostendorp 2003, Silverman 2011; see Reiss 2025 for discussion).
- We now use these tools to reanalyze cases of segment deletion with putative exceptions under the hypothesis that children acquiring language are *epistemically bounded* (in the sense of Fodor 1980:33f.) to analyze segments behaving differently in the same contexts as underlyingly distinct.

(97) Deletion rules and Inkelas' Protocol:

It turns out that Turkish has velars that delete intervocalically and velars that do not delete. Delete the rich forced us to posit that the velars that delete can be targetted as a natural class. This means that they must be *more* specified than the non-deleting ones. In other words, /k/ deletes, but /K/ does not. Similarly, Hungarian has two *h*'s, one that deletes in coda and one that does not. We have already seen /H/, which is partially underspecified—this one does not delete, but there is also an /h/ which does delete. See Gorman and Reiss (2025).

- Point 1: Set theory—an empty set is not nothing. Deleting a segment is not the same as deleting features.
- Point 2: Inkelas' Protocol fails utterly. Hungarian /H/ and Turkish /K/ do not alternate, but must be underspecified; Hungarian /h/ and Turkish /k/ do alternate and outcomes are phonologically predictable, but they are fully specified.

14 Catalan

Catalan (data from Wheeler 2005 and Kenstowicz and Kisseberth 1979) appears to have rules deleting /n/ and /r/. One rule deletes both of these −Lateral, +Coronal, +Sonorant segments when they occur word-finally. This data also shows deletion of /r/, but not /n/, before /s/—see the masc. plural forms.

(98) Basic *r* and *n* deletion:

Masc Sg	Masc Pl	Fem Sg	Fem Pl	Lexical Form	Gloss
bo	bons	bonə	bones	/bon-/	good
ple	plens	plenə	plenes	/plen-/	full
kla	klas	klarə	klares	/klar-/	plain
du	dus	durə	dures	/dur-/	hard
mal	mals	malə	males	/mal-/	bad
ultim	ultims	ultimə	ultimes	/ultim-/	last

The forms in (99) show that it is not the case that a single rule is responsible for deleting /r/ in codas—deletion of /r/ in [kla] and [klas] is due to separate rules. There is no general coda deletion rule for /r/ given, the fact that [r] surfaces in [for, fors, fortə, fortes]. The data also makes it clear that the deletion of /t/ and /d/ counterfeeds the deletion of /r/ before /s/, as in [fors]—if /t/ deleted before the rule deleting /r/ before /s/, then [fors] would not surface.

(99) Counterfeeding of *r* and *n* deletion by *t, d* deletion:

	Masc Sg	Masc Pl	Fem Sg	Fem Pl	Lexical Form	Gloss
a.	profun	profuns	profundə	profundes	/profund-/	deep
	for	fors	fortə	fortes	/fort-/	strong
	al	als	altə	altes	/alt-/	tall

(100) 'Exceptions' to *r* and *n* deletion:

Masc Sg	Masc Pl	Fem Sg	Fem Pl	Lexical Form	Gloss
pur	purs	purə	pures	/puR/	pure
kon	kons			/koN/	cone
tren	trens			/treN/	train

The solution that follows most naturally from LP assumptions and yields the simplest grammar is one in which the non-alternating segments correspond to a subset of the features of the alternating ones. The rule deleting /r/ and /n/ word-finally targets the natural class defined by the intersection /r/ ∩ /n/, the class of −Lateral coronal sonorants (so not including /l/). This set of features is denoted by $\mathscr{R}$ which includes, among others, −Lateral, +Coronal and +Sonorant, but no specification for Nasal. This natural class is denoted by [($\mathscr{R}$)] in Rule 101.

(101) Catalan word-final coronal sonorant deletion targets −Lateral segments:

Circled notation version

$$\overset{\text{X}}{[\textcircled{\mathscr{R}}]} \quad \mapsto \quad \epsilon \quad / \_\_ \%$$

Feature version

$$\overset{\text{X}}{\begin{bmatrix} -\textsc{Lateral} \\ +\textsc{Coronal} \\ +\textsc{Sonorant} \end{bmatrix}} \quad \mapsto \quad \epsilon \quad / \_\_ \%$$

$\mathscr{R}$ includes −Lateral, so Rule 101 targets /r/ and /n/, but not /l/. We propose that −Lateral is also lacking in the "exceptional", non-deleting consonants, the root-final sonorants in (100). Those segments are /R/ and /N/, unspecified for Lateral, but otherwise identical to /r/ and /n/, respectively. So Rule 101 does not target /R/ or /N/ (or /l/), which all lack −Lateral.

Rule 102 deletes /r/, but not /R/, before /s/. This is possible, again, because /r/ is more highly specified than /R/. To be precise, the rule targets coronal sonorants that are specified as −Lateral and −Nasal, so only the single segment /r/.

(102) Catalan deletion of /r/ before /s/:

Circled notation version

$$\overset{\text{X}}{[\textcircled{r}]} \quad \mapsto \quad \epsilon \quad / \_\_ [\textcircled{s}]$$

Feature version

$$\overset{\text{X}}{\begin{bmatrix} -\textsc{Lateral} \\ -\textsc{Nasal} \\ +\textsc{Coronal} \\ +\textsc{Sonorant} \end{bmatrix}} \quad \mapsto \quad \epsilon \quad / \_\_ [\textcircled{s}]$$

Since undeleted /r/ and /n/ neutralize with /R/ and /N/, repsectively, we posit a default unification Rule 103 adding −Lateral to all coronal sonorants. The rule target is the smallest natural class containing /R/ and /N/, which is defined using the features in /R/ ∩ /N/ = /$\mathscr{N}$/, so the class is [$\textcircled{\mathscr{N}}$]. This natural class contains /r, n, R, N/, by specificity, but Rule 103 applies vacuously to the first two of these segments, because of vacuous unification—they are underlyingly already specified as −Lateral. The segment /l/ will be unaffected because of unification failure.

(103) Catalan default addition of −Lateral:

Circled notation version

$$[\textcircled{\mathscr{N}}] \quad \sqcup \quad \{-\textsc{Lateral}\}$$

Feature version

$$\begin{bmatrix} +\textsc{Coronal} \\ +\textsc{Sonorant} \end{bmatrix} \quad \sqcup \quad \{-\textsc{Lateral}\}$$

Let's recap the differences among /R/, /N/, /$\mathscr{R}$/, and /$\mathscr{N}$/.

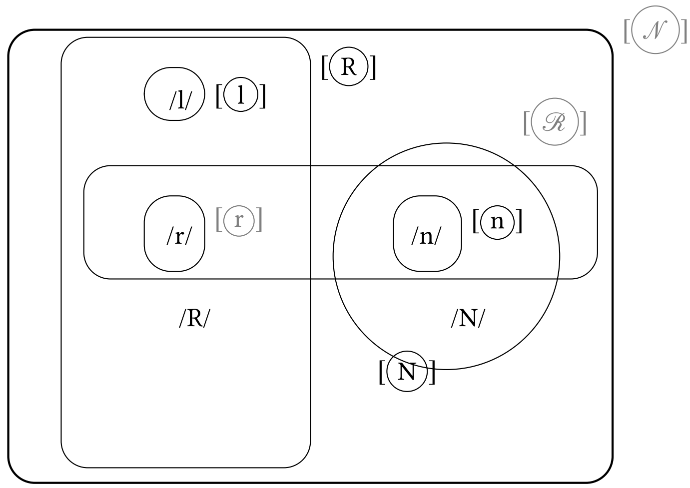
(104) Partial feature specifications w/ lexical segments in / /:

	/r/	/R/	/n/	/N/	$\mathscr{R}$	$\mathscr{N}$	/l/
Nasal	−	−	+	+			−
Lateral	−		−		−		+
Sonorant	+	+	+	+	+	+	+
Coronal	+	+	+	+	+	+	+

- [$\textcircled{\mathscr{R}}$] = [−Lateral, +Sonorant, +Coronal] = {r, n }
- [$\textcircled{R}$] = [−Nasal,+Sonorant, +Coronal] = {r, R, l}
- [$\textcircled{\mathscr{N}}$] = [+Sonorant, +Coronal] = {r, n, l, R, N }

Rules 101-103 target the natural classes in gray, [$\textcircled{\mathscr{R}}$], [$\textcircled{r}$] and [$\textcircled{\mathscr{N}}$], respectively. The diagram in (105) gives an indication of the apparent complexity of the interactions, but in LP, each rule merely targets explicitly defined natural classes. The complexity is *only* apparent. Each term expressed with square brackets, like [$\textcircled{N}$], labels a region. Actual segments of Catalan are written inside of back slashes, like /N/.

(105) Euler diagram of some Catalan natural classes based on the set of segments {r, R, n, N l}:



Catalan punchline: There are no exceptions in Catalan. LP allows us to reduce the patterns to pure phonology, as in the previous languages examined. All we need is the three operators and specificity.

15 Conclusions and Future plans

- Consistent logic is useful
- An austere system can be powerful
- Cyclic rule application in LP Kasenov and Reiss (2025)]
- LP rules for the change component of morpho-phonological rules
- Alternative account of Welsh soft mutation? (cf. Hammond et al. 2020)
  - No mutation: b~ b /b/
  - Mutation: b~ v /B/
  - No mutation: g~ g /G/
  - Mutation: g~ ϵ /g/

References

Appelbaum, Irene. 1996. The lack of invariance problem and the goal of speech perception. In *The 4th International Conference on Spoken Language Processing*, 1541–1544.

Archangeli, Diana, and Douglas Pulleyblank. 1994. *Grounded Phonology*. MIT Press.

Bale, Alan, and Charles Reiss. 2018. *Phonology: A Formal Introduction*. MIT Press.

Bayer, Sam, and Mark Johnson. 1995. *Features and Agreement: Proceedings of the 33rd Annual Meeting of the Association for Computational Linguistics*. San Francisco, CA: Morgan Kaufmann.

Benz, Johanna, and Veno Volenec. 2023. Two logical operations underlie all major types of segmental alternations. Paper presented at the 30th Manchester Phonology Meeting.

Buckley, Eugene. 1994. Prespecification of default features: The two /i/'s of Kashaya. In *NELS 24: Proceedings of Twenty-Fourth Annual Meeting of the North East Linguistic Society*, 17–30.

Calabrese, Andrea. 1985. Metaphony in Salentino. *Rivista di Grammatica Generativa* 10:1–140.

Chomsky, Noam. 1957. Review of Jakobson-Halle, 1956. *International Journal of American Linguistics* 23:234–242.

Chomsky, Noam, and Morris Halle. 1965. Some controversial questions in phonological theory. *Journal of Linguistics* 1:97–138.

Chomsky, Noam, and Morris Halle. 1968. *The Sound Pattern of English*. Harper & Row.

Dabbous, Rim, Marjorie Leduc, Charles Reiss, and David Ta-Chun Shen. 2024. Locality is epiphenomenal: Adjacency is opaqueness. In *Proceedings of the 2024 annual conference of the Canadian Linguistic Association*.

Davenport, Michael, and S. J Hannahs. 2010. *Introducing phonetics and phonology*. London: Hodder Education, 3rd ed edition.

Dresher, B. Elan. 2009. *The Contrastive Hierarchy in Phonology*. Cambridge University Press.

Fodor, Jerry. 1980. Reply to Putnam. In *Language and Learning: The Debate between Jean Piaget and Noam Chomsky*, ed. Massimo Piattelli-Palmarini, 325–334. Harvard University Press.

Frantz, Donald G. 2017. *Blackfoot Grammar*. University of Toronto Press, 3rd edition.

Fudge, E. C. 1967. The nature of phonological primes. *Journal of Linguistics* 3:1–36.

Gallistel, C. Randy, and Adam Philip King. 2009. *Memory and the computational brain: Why cognitive science will transform neuroscience*. Chichester: Wiley-Blackwell.

Gorman, Kyle. to appear. Greek fortition in logical phonology. *Canadian Journal of Linguistics/Revue canadienne de linguistique* .

Gorman, Kyle, and Charles Reiss. 2025. Natural class reasoning in segment deletion rules. NELS talk.

Halle, Morris, and George N Clements. 1983. *Problem book in phonology: a workbook for introductory courses in linguistics and in modern phonology*. MIT Press.

Hammond, Michael, Elise Bell, Skye Anderson, Peredur Webb-Davies, Diane Ohala, Andrew Carnie, and Heddwen Brooks. 2020. Category-specific effects in welsh mutation. *Glossa* .

Harms, Robert T. 1968. *Introduction to phonological theory*. Englewood Cliffs, NJ: Prentice-Hall.

Harris, John. 1994. *English sound structure*. Oxford: Blackwell Publishers.

Hayes, Bruce. 2009. *Introductory Phonology*. Blackwell.

Hayward, Dick. 1984. *The Arbore language: a first investigation, including a vocabulary*, volume 2. John Benjamins Publishing Company.

Hualde, José Ignacio. 1991. *Basque phonology*. Routledge.

Inkelas, Sharon. 1995. The consequences of optimization for underspecification. In *Proceedings of the North East Linguistic Society 25*, 287–302.

Inkelas, Sharon, and Young-Mee Yu Cho. 1993. Inalterability as prespecification. *Language* 69:529–574.

Kaplan, Lawrence D. 1981. *Phonological Issues in North Alaskan Inupiaq*. Alaska Native Language Center.

Kasenov, Daniar, and Charles Reiss. 2025. Unification failure and cycles in Logical Phonology: An account of voicing alternations in Polish and English. ms.

Kenstowicz, Michael, and Charles Kisseberth. 1979. *Generative phonology: Description and theory*. New York: Academic Press.

Kornai, András. 2008. *Mathematical linguistics*. London: Springer.

Lees, Robert B. 1961. *The Phonology of Modern Standard Turkish*. Indiana University Publications.

Lightner, Theodore M. 1963. A note on the formulation of phonological rules. Technical Report 68.

Matamoros, Camila, and Charles Reiss. 2016. Symbol taxonomy in biophonology. *Biolinguistic Investigations on the Language Faculty* 235:41.

McCarthy, John J. 2008. The gradual path to cluster simplification. *Phonology* 25:271–319.

Mielke, Jeffrey. 2008. *The emergence of distinctive features*. Oxford: Oxford University Press.

Odden, David. 2013. Formal phonology. *Nordlyd* 40:249–273.

van Oostendorp, Marc. 2003. Schwa in phonological theory. In *The Second Glot International State-of-the-Article Book: The Latest in Linguistics*, ed. Lisa Cheng and Rint Sybesma, 431–461. De Gruyter Mouton.

Poĺomská, Anna. 2025. Phonological relations between palatalizers and the phonemic system: A case study on czech. *LingBaW. Linguistics Beyond and Within* 11:171–205.

Poser, William. 1982. Phonological representation and action-at-a-distance. In *The Structure of Phonological Representations*, ed. Harry van der Hulst and Norval Smith, 121–58. Foris.

Reiss, Charles. 2003. Deriving the feature-filling/feature-changing contrast: An application to Hungarian vowel harmony. *Linguistic Inquiry* 34:199–224.

Reiss, Charles. 2025. Delete the rich: On the non-existence of 'weak' schwa deletion. Ms. LOA-006. URL: `https://lingbuzz.net/lingbuzz/008747`.

Reiss, Charles, and Veno Volenec. 2022. Conquer primal fear: Phonological features are innate and substance free. *Canadian Journal of Linguistics* 67:581–610.

Shieber, Stuart. 1986. *An Introduction to Unification-Based Approaches to Grammar*. CSLI Publications.

Silverman, Daniel. 2011. Schwa. In *Blackwell Companion to Phonology*, ed. Marc van Oostendorp, Colin J. Ewen, Beth Hume, and Keren Rice, chapter 26. Wiley Online Library.

Siptár, Péter, and Miklós Törkenczy. 2000. *The Phonology of Hungarian*. Oxford University Press.

Spencer, Andrew. 1996. *Phonology: Theory and description*. Oxford: Blackwell.

Stevens, Kenneth N. 2000. *Acoustic phonetics*. MIT Press.

Volenec, Veno, and Charles Reiss. 2020. Formal generative phonology. *Radical: A Journal of Phonology* 2:1–148.

Volenec, Veno, and Charles Reiss. 2025. Cognitive phonetics: The universal phonology-phonetics interface. In *Cambridge Handbook of Linguistic Interfaces*, ed. Antonio Fábregas, Laia Mayol, and Yanina Prystauka, to appear. Cambridge University Press.

Wheeler, Max W. 1972. Distinctive features and natural classes in phonological theory1. *Journal of Linguistics* 8:87–102.

Wheeler, Max W. 2005. *The phonology of Catalan*. OUP Oxford.