Compiling Search & Change Rules into Subsequential Finite-State Transducers

Malek Azadegan[1]

[1] *The Graduate Center, CUNY*

Introduction. Phonological computation refers to the processes by which words' internal representations are transformed into the external forms that are actually pronounced (Chomsky and Halle, 1968; Kenstowicz and Kisseberth, 1979). Search & Change (S&C) is "a model of phonological computation based on rules characterized by a small set of parameters" (Dabbous et al., 2021). S&C operates on segments within words, using Logical Phonology's formal definitions of valued features, segments and natural classes.

Features and segments. Under Logical Phonology (LP), Universal Grammar provides binary feature values $\mathcal{W} = \{+, -\}$ and a finite feature set $\mathcal{F}$, so features are members of $\mathcal{W} \times \mathcal{F}$ (e.g. $+\textsc{f}$) (Dabbous et al., 2025). Segments are sets of such specifications, e.g.:

(1) $/\text{u}/ = \{+\textsc{Syllabic}, +\textsc{Sonorant}, +\textsc{Back}, +\textsc{Round}, +\textsc{High}, -\textsc{Low}, \ldots\}$

A segment is said to be *consistent* if and only if it does not contain both $+\textsc{f}$ and $-\textsc{f}$ for any feature $f \in \mathcal{F}$ (Dabbous et al., 2025). Underspecified segments are licit in LP, and are denoted using capital letters (i.e., /U/ can describe the segment /u/ that is underspecified for any feature $\textsc{f} \in /\text{u}/$).

Natural classes. Phonological processes often target groups of segments that share certain properties, known as natural classes. Rather than computing minimal feature sets, natural classes are defined using maximal feature intersection (Gorman and Reiss, 2023). For instance, take the definition of /u/ from (1) and the definition $/\text{i}/ = \{+\textsc{Syllabic}, +\textsc{Sonorant}, -\textsc{Back}, -\textsc{Round}, +\textsc{High}, -\textsc{Low}, \ldots\}$. Their shared specification is given by feature intersection:

(2) $/\text{u}/ \cap /\text{i}/ = \{+\textsc{Syllabic}, +\textsc{Sonorant}, +\textsc{High}, -\textsc{Low}\}$

Using bracket notation (Bale and Reiss, 2015), a natural class containing /u/ and /i/ can be defined as:

(3) $[+\textsc{Syllabic}, +\textsc{Sonorant}, +\textsc{High}, -\textsc{Low}] = \{x \mid x \supseteq \{+\textsc{Syllabic}, +\textsc{Sonorant}, +\textsc{High}, -\textsc{Low}\}\}$

This approach may result in a broader set than originally intended, which is an expected consequence of defining classes based only on shared features (Gorman and Reiss, 2023).

The universal natural class $[\,] = \Sigma$ targets all segments, since every segment is a superset of the empty set.

Words. A word is a finite sequence of segments $\langle s_0, \ldots, s_{n-1} \rangle$, bounded by ⋉ and ⋊, where each segment is a set of valued features, conventionally notated using IPA symbols (e.g. *cat*: $\langle ⋉, \text{k}, \text{æ}, \text{t}, ⋊ \rangle$).

Operations on segments. Structuring segments as sets allows set-theoretic operations to be used to transform them. Feature deletion uses ordinary set subtraction, while feature-filling processes use unification ($\sqcup$), introduced by Bale et al. (2014), which performs a union only when no contradictory feature values are introduced (i.e., no segment should be simultaneously $+\textsc{f}$ and $-\textsc{f}$ for any feature in $\mathcal{F}$). We adopt the definition from Gorman and Reiss (2025), which captures the atomicity of the unification operation:

(4) $A \sqcup B = A \cup c\text{F} \mid c\text{F} \in B \wedge -c\text{F} \notin A$

Search & Change. A S&C rule is a 5-tuple ⟨Inr, Trm, Out, Dir, Cnd⟩. For each s_i in a word w, if $s_i \in \textsc{Inr}$ (initiator), a directional search (denoted by Dir) finds the nearest $s_j \in \textsc{Trm}$ (terminator) from s_i; if $s_j \in \textsc{Cnd}$ (i.e., if s_j licenses a transformation), the output function Out applies to s_i (Dabbous et al., 2021, 2024) . Formally, search can be defined as:

$$\Theta(\textsc{Dir}, \textsc{Trm}, i) = \begin{cases} \min\{j \mid j > i \wedge s_j \in \textsc{Trm}\} & \text{if } \textsc{Dir} = \text{R} \ \wedge \ \exists j \mid j > i \wedge s_j \in \textsc{Trm}, \\ \max\{j \mid j < i \wedge s_j \in \textsc{Trm}\} & \text{if } \textsc{Dir} = \text{L} \ \wedge \ \exists j \mid j < i \wedge s_j \in \textsc{Trm}, \\ \downarrow & \text{otherwise.} \end{cases}$$

Different choices of $\textsc{Inr}$, $\textsc{Trm}$, $\textsc{Dir}$, $\textsc{Out}$, and $\textsc{Cnd}$ allow S&C rules to model a wide range of phonological phenomena. For instance, setting either $\textsc{Inr}$ or $\textsc{Trm}$ to the universal natural class makes every segment a search initiator or terminator, respectively. This captures processes that apply to all segments in a word, or that require adjacency to a particular class of segment.

The output function $\textsc{Out}$ is a composition of subtraction and unification operations, each of type $S \times S \to S$, meaning they take a segment and a feature bundle as input, and return a (possibly modified) segment. These can be used with a static feature bundle to produce $\textsc{out}$ functions of type $S \to S$. Formally:

$$\textsc{Subtract}(s_i, f) : S \times S \to S = s_i \setminus f \qquad \textsc{Unify}(s_i, f) : S \times S \to S = s_i \sqcup f$$

The S&C specification can then be defined as follows (where $\downarrow$ means *is defined*):

$$s'_i(\textsc{Rule}) = \begin{cases} \textsc{Out}(s_i) & \text{if } s_i \in \textsc{Inr} \wedge \Theta(\mathrm{R}, \textsc{Trm}, i) \downarrow \wedge\, s_{\Theta(\mathrm{R},\textsc{trm},i)} \in \textsc{Cnd} \\ s_i & \text{otherwise} \end{cases}$$

Given a word $w = \langle s_0, s_1, \ldots, s_{n-1}\rangle$, the transduced output is $w' = \langle s'_0, s'_1, \ldots, s'_{n-1}\rangle$. I adopt the simplifying assumption that rules apply simultaneously, as mentioned by Dabbous et al. (2021). Thus, each rule applies globally, without directional iterativity.

Example Rules. Assume that the English plural suffix /-S/ corresponds to the segment /S/ = /s/ ∩ /z/, which is underspecified for $\textsc{Voice}$. Voicing is assigned by feature-filling rules that copy the $\textsc{Voice}$ value of the immediately preceding segment.

All rules initiate searches from segments that are supersets of /S/ and search leftward, so we define:

$$\textsc{Inr}_{plural} = [\textsc{f} \mid \textsc{f} \in /\mathrm{S}/] \qquad \textsc{Dir} = \mathrm{L}$$

We split the voicing rule into two, one applying when the triggering segment is $+\textsc{Voice}$ and one when it is $-\textsc{Voice}$. This requires two output functions:

$$\textsc{Out}_{\phi}(s) = \textsc{Unify}(s, \{+\textsc{Voice}\}) \qquad \textsc{Out}_{\psi}(s) = \textsc{Unify}(s, \{-\textsc{Voice}\})$$

We also define two corresponding licensing conditions:

$$\textsc{Cnd}_{\phi} = [+\textsc{Voice}] \qquad \textsc{Cnd}_{\psi} = [-\textsc{Voice}]$$

The complete S&C analysis consists of the following two rules:

$\textsc{Rule}_1$: $\langle \textsc{Inr}_{plural},\ \textsc{Trm} = [\,],\ \textsc{Out}_{\psi},\ \textsc{Dir} = \mathrm{L},\ \textsc{Cnd}_{\psi}\rangle$ *Plural voicing after* $-$voice
$\textsc{Rule}_2$: $\langle \textsc{Inr}_{plural},\ \textsc{Trm} = [\,],\ \textsc{Out}_{\phi},\ \textsc{Dir} = \mathrm{L},\ \textsc{Cnd}_{\phi}\rangle$ *Plural voicing after* $+$voice

Application of these rules yields the following mappings:

1. $\langle ⋉, \mathrm{k}, \text{æ}, \mathrm{t}, \underline{\mathrm{S}}, ⋊\rangle \rightsquigarrow \langle ⋉, \mathrm{k}, \text{æ}, \mathrm{t}, \underline{\mathrm{s}}, ⋊\rangle$ ($\textsc{Rule}_1$ applies, $\textsc{Rule}_2$ has no effect)
2. $\langle ⋉, \mathrm{b}, \text{æ}, \mathrm{g}, \underline{\mathrm{S}}, ⋊\rangle \rightsquigarrow \langle ⋉, \mathrm{b}, \text{æ}, \mathrm{g}, \underline{\mathrm{z}}, ⋊\rangle$ ($\textsc{Rule}_2$ applies, $\textsc{Rule}_1$ has no effect)
3. $\langle ⋉, \mathrm{l}, \varepsilon, \mathrm{n}, \underline{\mathrm{z}}, ⋊\rangle \rightsquigarrow \langle ⋉, \mathrm{l}, \varepsilon, \mathrm{n}, \underline{\mathrm{z}}, ⋊\rangle$ (No rule applies)

Implications. The S&C specification, taken literally, has quadratic time complexity, but the search can be reformulated as a single pass over the word, yielding a linear-time algorithm. Since $\textsc{inr}$, $\textsc{trm}$, $\textsc{cnd}$, and $\textsc{out}$ all involve finite sets, these operations can be fully hardcoded into FST transitions and states. The resulting transducers are subsequential (i.e., fully deterministic and streaming) meaning phenomena modeled by S&C are within the class learnable from positive input–output pairs by algorithms such as OSTIA (Mohri, 1997; Heinz, 2018; Oncina et al., 1993), which works by building a prefix tree transducer from input–output examples and incrementally merging states while preserving determinism, generalizing beyond observed data.

Keywords: Search & Change, finite state transducer, phonology, learnability

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