Formal languages I: practicum problems

Problem For any arbitrary set S:

- Is S a member of $\{S\}$?

- Is $\{S\}$ a member of $\{S\}$?

- $\{S\}$ a subset of $\{S\}$?

Solution

- $S \in \{S\}$

- $\{S\} \notin \{S\}$

- $\{S\} \subseteq \{S\}$

Problem Let:

$$A = \{\{\varnothing\}, \{X\}, X\}$$
$$B = X$$
$$C = \{X\}$$
$$D = \{\{X\}\}$$
$$E = \{\{X\}, X\}$$
$$F = \varnothing$$
$$G = \{\varnothing\}$$
$$H = \{\{\varnothing\}\}$$
$$I = \{\varnothing, \{\varnothing\}\}$$

Of these sets:

- Which are members of A?

- Which are subsets of A?

- Which are members of I?

- Which are subsets of I?

Solution

- $B, C, G \in A$

- $A, C, D, E, F, H \subseteq A$

- $F, G \in I$

- $I, F, G, H \subseteq I$

Problem   Let:

$$A = \{\mathtt{ab}, \mathtt{cde}, \mathtt{fghi}\}$$
$$B = \{\mathtt{j}, \mathtt{kl}\}$$
$$C = \{\mathtt{mn}\}$$

Now, list all the strings in the following (finite) languages:

- AC

- $A \cup C$

- $B^R \cup A$

- $C^? C^?$

Solution

- $AC = \{\mathtt{abmn}, \mathtt{cdemn}, \mathtt{fghimn}\}$

- $A \cup C = \{\mathtt{ab}, \mathtt{cde}, \mathtt{fghi}, \mathtt{mn}\}$

- $B^R \cup A = \{\mathtt{j}, \mathtt{lk}, \mathtt{ab}, \mathtt{cde}, \mathtt{fghi}\}$

- $C^? C^? = \{\epsilon, \mathtt{mn}, \mathtt{mnmn}\}$

Problem   Let:

$$V = \{\mathtt{Saruman}, \mathtt{Sauron}, \mathtt{Shelob}\}$$

List some strings in the set V^*.

Solution   This set V^* is infinite and consists of the concatenation of zero or more strings in V. Here are some members of this set:

- ϵ

- Saruman

- Sauron

- Shelob

- SarumanSaruman

- SarumanSauron

- SarumanShelob

- SauronShelob

- ShelobSaruman

- ShelobShelobShelob

- ...

Problem   Let:

$$E = \{\texttt{Elrond}, \texttt{Elros}\}$$
$$H = \{\texttt{Bilbo}, \texttt{Frodo}\}$$

List the strings in $E - H$.

Solution $E - H$ is the set of strings in E that are not in H. Since E and H have no common elements, $E - H = E = \{\texttt{Elrond}, \texttt{Elros}\}$.

Problem   List all the strings matched by the following (finite) regular expression:

```
/(Godzilla|Kong)( (: King of the Monsters|: Skull Island|
              vs. (Goldzilla|Kong)))?/
```

Solution All the following are matched, in no particular order:

- `Godzilla`

- `Godzilla: King Of the Monsters`

- `Godzilla: Skull Island`

- `Godzilla vs. Godzilla`

- `Godzilla vs. Kong`

- `Kong`

- `Kong: King Of the Monsters`

- `Kong: Skull Island`

- `Kong vs. Godzilla`

- `Kong vs. Kong`

Problem List all the strings matched by the following (finite) regular expression:

```
/I am a ((proud )?wizard|baboon( brandishing a stick)?|
                  (fashionable )?cat)/
```

Solution

- `I am a wizard`

- `I am a proud wizard`

- `I am a baboon`

- `I am a baboon brandishing a stick`

- `I am a cat`

- `I am a fashionable cat`

Problem Which regular expression matches both `mother` and `father` and nothing else in the following sentence?

```
              Your mother was a hamster
        and your father smelt of elderberries.
```

1. `/.+ther/`

2. `/\w+ther/`

3. `/\w{2}[ther]/`

4. `/mo|fa(ther)/`

Solution Regular expression #2 matches both `mother` and `father`.

Problem Which regular expression matches just the words with two identical adjacent characters in the following sentence?

```
      You must cut down the mightiest tree in the forest
                    with a herring!
```

1. `/\w*(\w)\1\w*/`

2. `/\w*(\w)+\w*/`

3. `/\w*\w{2}\w*/`

4. `/\w*[\w]{2}\w*/`

Solution Regular expression #1 matches both `tree` and `herring`. NB: the `\1` portion, known as a *backtrace*, is not part of the definition of regular languages, though it is supported by most regular expression engines, including Python's `re`.

Problem Which regular expression matches only the bold portion of the following sentence?

```
    What sad times are these when passing ruffians can say
              `%&$#@!'  at will to old ladies.
```

1. `/[^A-z\s.`']+/`

2. `/^(A-z\s.`')+/`

3. `/[\W\S]+!/`

4. `/[^A-z.`']?/`

Solution Regular expression #1 matches the bold portion.

Problem Write a regular expression that matches `I am a hobbit from`, followed by zero or more occurrences of either `the Shire` or `Galt's Gulch`, followed by `who is home` and optionally `sick`. Handle spacing between words using your knowledge of English.

Solution

```
/I am a hobbit from(( the Shire| Galt's Gulch)*)
            who is home(sick)?/
```